Chapter 7

Kinetic Energy & Work

7.2 What is energy?

One definition:

Energy is a scalar quantity associated with the state (or condition) of one or more objects.

Some characteristics:

1. Energy can be transformed from one type to another and transferred from one object to another.
2. The total amount of energy is always the same (energy is ***conserved****).*

7.3 Kinetic Energy

- **Kinetic energy *K*** is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy.

- For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \tfrac{1}{2}mv^2 \qquad \text{(kinetic energy).}$$

- The SI unit of kinetic energy (and every other type of energy) is the **joule (J):**
  - **1 J = 1 kgm^2/s^2**

Example: kinetic energy

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant 0.26 m/s^2, what was the total kinetic energy of the two locomotives just before the collision?

Fig. 7-1 The aftermath of an 1896 crash of two locomotives. *(Courtesy Library of Congress)*

Calculations: We choose Eq. 2-16 because we know values for all the variables except v:

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3$ m (half the initial separation), this yields

$$v^2 = 0 + 2(0.26 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}),$$

or

$$v = 40.8 \text{ m/s}$$

(about 150 km/h).

We can find the mass of each locomotive by dividing its given weight by g:

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

Now, using Eq. 7-1, we find the total kinetic energy of the two locomotives just before the collision as

$$K = 2(\tfrac{1}{2}mv^2) = (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2$$
$$= 2.0 \times 10^8 \text{ J}. \qquad \text{(Answer)}$$

This collision was like an exploding bomb.

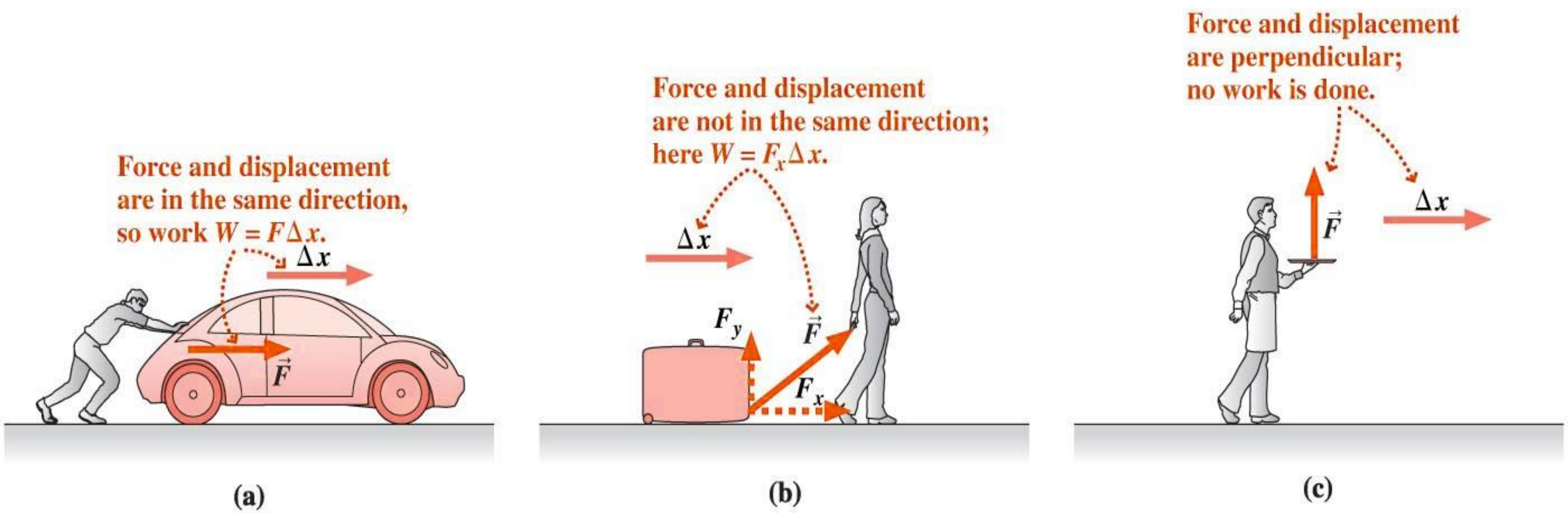
7.4: Work

Work (W): Energy transferred to or from an object by means of a mechanical force acting on the object.

Energy transferred to the object is positive work, and energy transferred from the object is negative work.

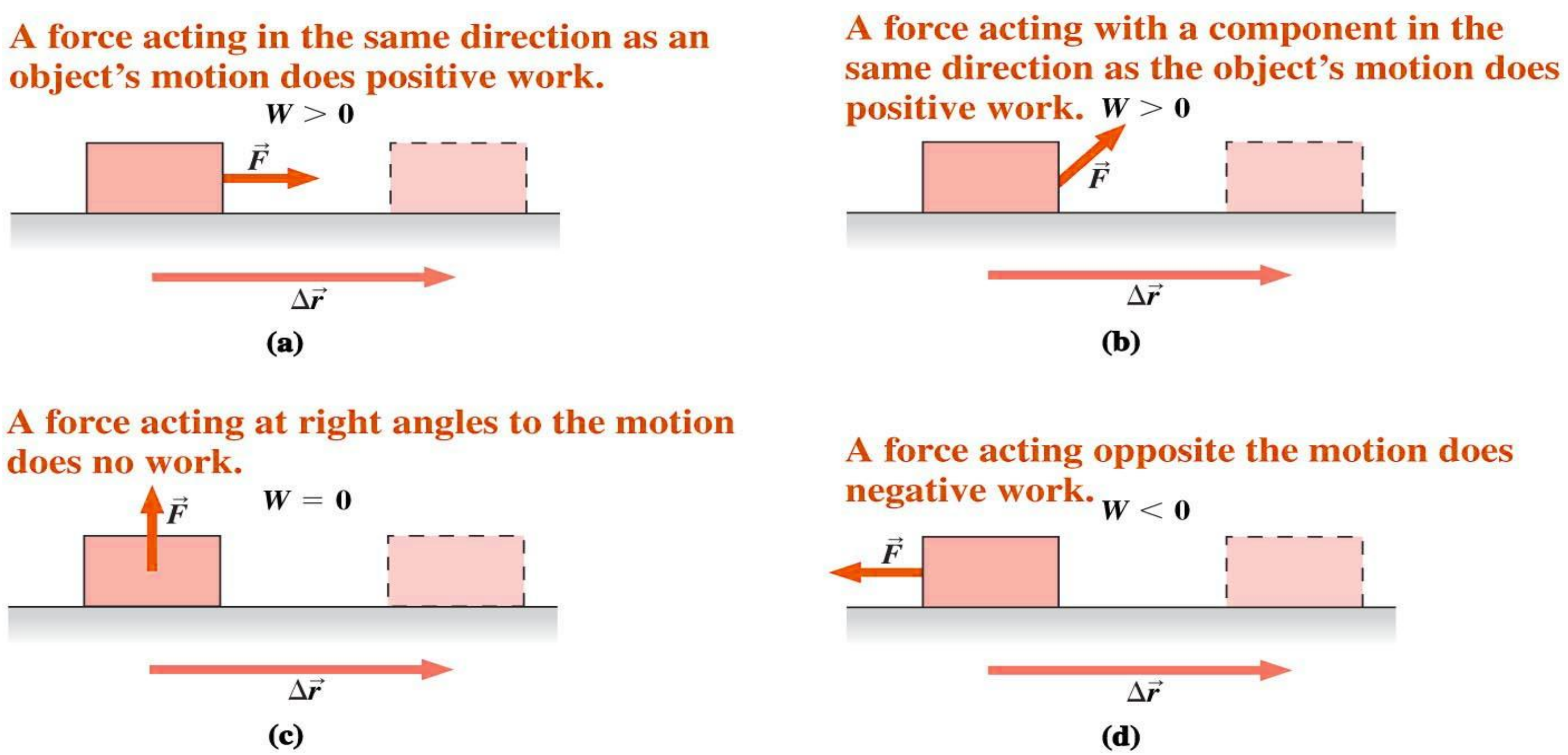
In one dimension: $$W = F_x \Delta x$$

More generally, work depends on the *component of force in the direction of motion*:



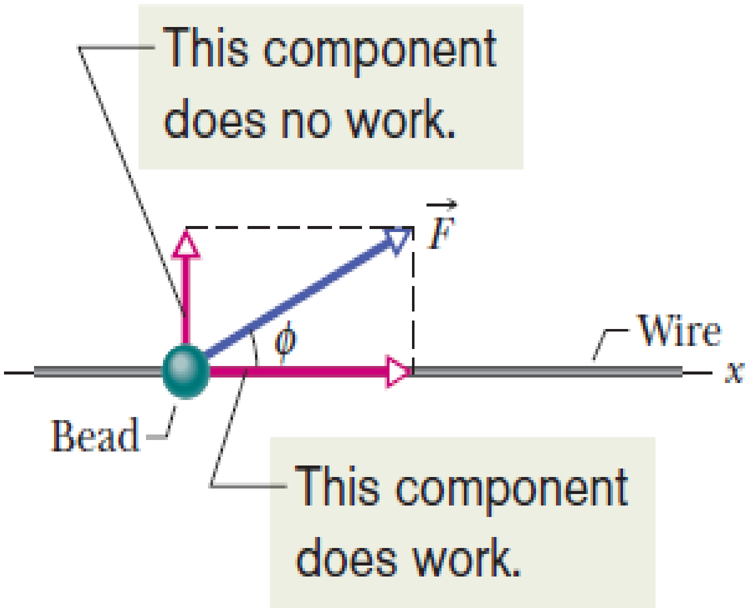
Work can be positive or negative:

- Work is positive if the force has a component in the same direction as the motion.
- Work is negative if the force has a component opposite the direction of motion.
- Work is zero if the force is perpendicular to the motion.



7.5: Work and Kinetic Energy (constant force, scalar product)

To calculate the work a force ***F*** does on an object as the object moves through some displacement ***d***, we use only the force component along the object's displacement. The force component perpendicular to the displacement direction does zero work.



For a *constant* force ***F***, the work done W is:

$$W = \vec{F}\cdot\vec{d} = Fd\cos\phi$$

A constant force directed at angle f to the displacement (in the x-direction) of a bead does work on the bead. The only component of force taken into account here is the x-component.

When two or more forces act on an object, the net work done on the object is the sum of the works done by the individual forces.

3.8: Multiplying vectors (scalar or dot product)

The scalar product between two vectors is written as:

$$\vec{a} \cdot \vec{b}$$

It is defined as:

$$\vec{a} \cdot \vec{b} = ab \cos \phi,$$

Here, a and b are the magnitudes of vectors **a** and **b** respectively, and Φ is the angle between the two vectors. The right hand side is a scalar quantity.

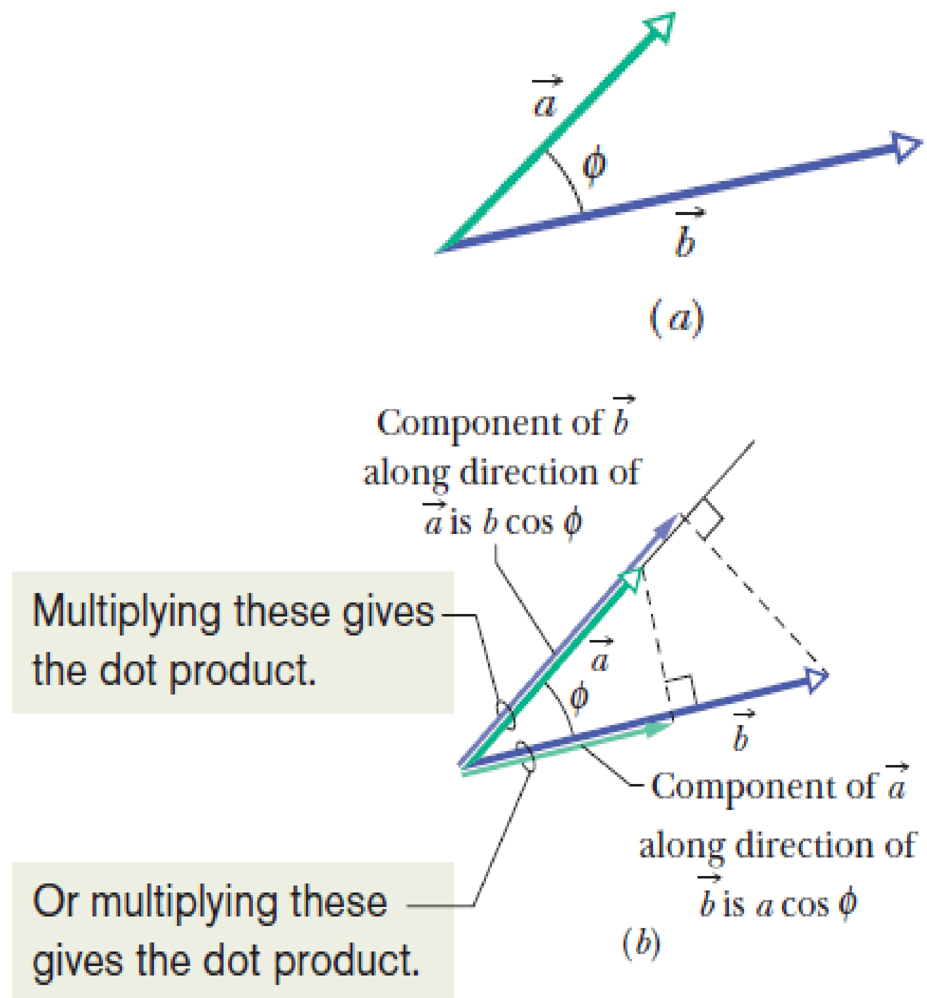


Fig. 3-18 (*a*) Two vectors $\vec{a}$ and $\vec{b}$, with an angle ϕ between them. (*b*) Each vector has a component along the direction of the other vector.

3.8: Multiplying Vectors (scalar or dot product)

Work is conveniently characterized using the *scalar (dot) product* -- a method of multiplying two vectors to produce a scalar that depends on the magnitude of the vectors & the angle between them.

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where A and B are the magnitudes of the vectors and θ is the angle between them.

***With vectors in component (unit vector) form, the scalar product can be written*:**

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- **Work is the scalar product of force with displacement:**

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

<u>**Work-Kinetic Energy Theorem**</u>:

- The change in kinetic energy of a particle is equal to the net work done on the particle.
- Net work is the work done by the net force acting on a particle.
- The work done is equal to the total energy transferred to the particle by means of mechanical forces.

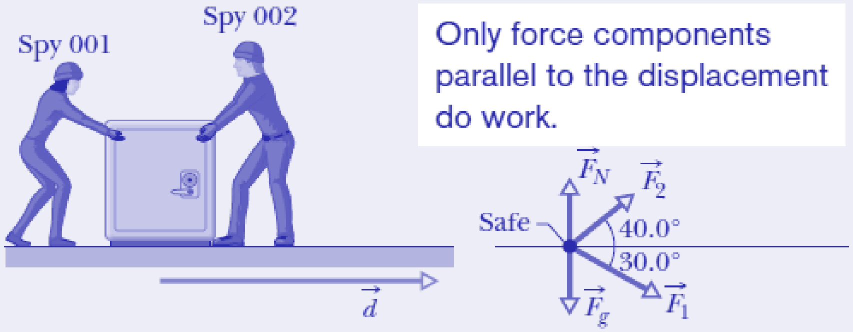
$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on} \\ \text{the particle}\end{pmatrix}$$

$$\boxed{\Delta K = W_{net}}$$

<u>Note</u>: The theorem holds true for both positive and negative work: If the net work done on a particle is positive, then the particle's kinetic energy increases by the amount equal to the work done (amount of energy transferred); the converse is also true.

Example: Industrial spies

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



(*a*) (*b*)

Only force components parallel to the displacement do work.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0 \text{ N})(8.50 \text{ m})(\cos 30.0°) = 88.33 \text{ J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos \phi_2 = (10.0 \text{ N})(8.50 \text{ m})(\cos 40.0°) = 65.11 \text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33 \text{ J} + 65.11 \text{ J} = 153.4 \text{ J} \approx 153 \text{ J}. \qquad \text{(Answer)}$$

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd \cos 90° = mgd(0) = 0 \qquad \text{(Answer)}$$

and

$$W_N = F_N d \cos 90° = F_N d(0) = 0. \qquad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4 \text{ J})}{225 \text{ kg}}} = 1.17 \text{ m/s}. \qquad \text{(Answer)}$$

Example: Constant force in unit vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

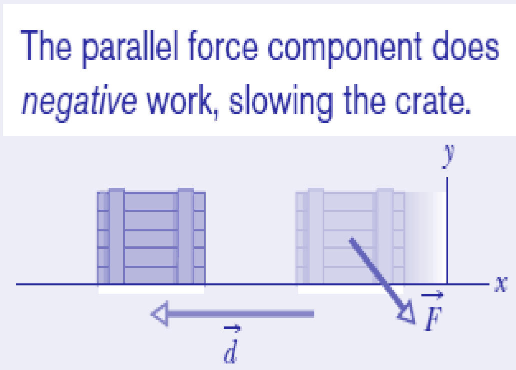


Fig. 7-5 Force $\vec{F}$ slows a crate during displacement $\vec{d}$.

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{i} + (-6.0\text{ N})\hat{j}]\cdot[(-3.0\text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$, and $\hat{k}\cdot\hat{k}$ are nonzero (see Appendix E). Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{i}\cdot\hat{i} + (-6.0\text{ N})(-3.0\text{ m})\hat{j}\cdot\hat{i}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}. \qquad \text{(Answer)}$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

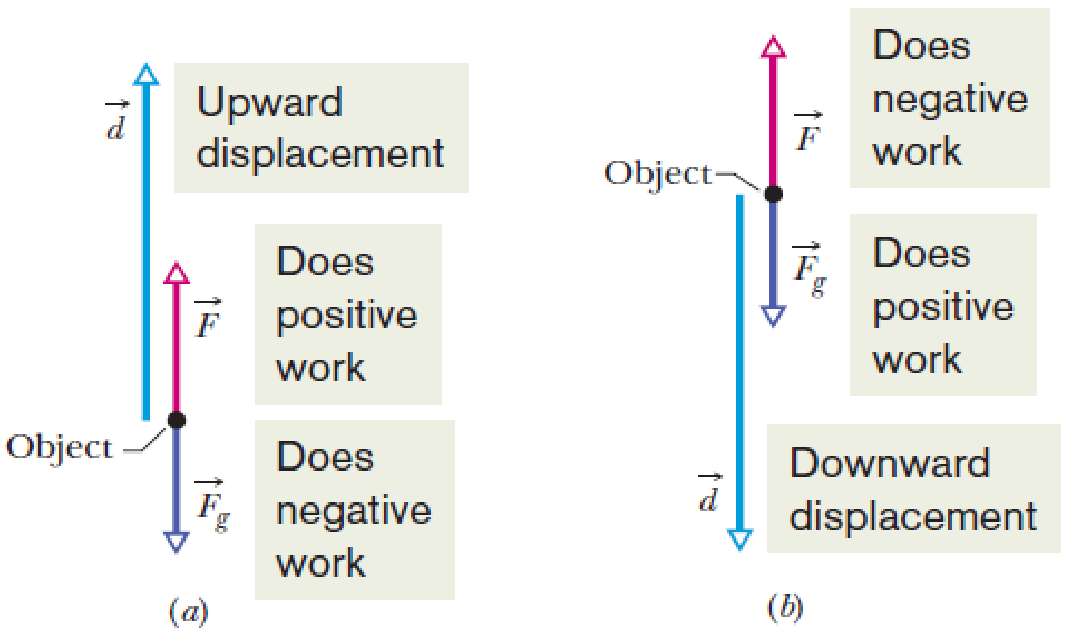
$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

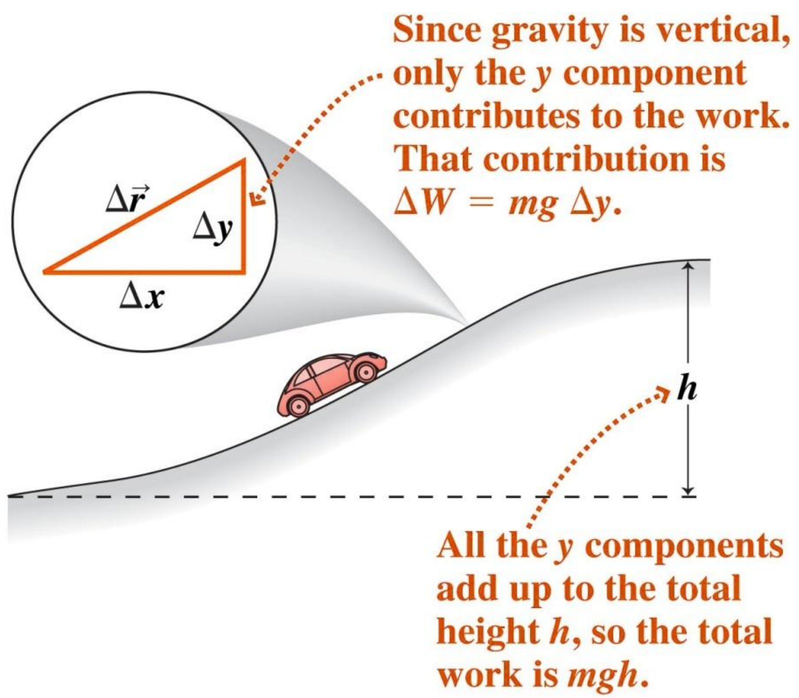
Less kinetic energy means that the crate has been slowed.

7.6: Work Done by Gravitational Force

$$W_g = mgd\cos\phi \qquad \text{(work done by gravitational force).}$$

$$W = mgh$$





(a) An applied force lifts an object. The object's displacement makes an angle Φ =180° with the gravitational force on the object. The applied force does positive work on the object.

(b) An applied force lowers an object. The displacement of the object makes an angle with the gravitational force. The applied force does negative work on the object.

Example: Accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8a).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

Calculation: From Fig. 7-8b, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then, from Eq. 7-12, we find

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1)$$
$$= 5.88 \times 10^4\text{ J} \approx 59\text{ kJ.} \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

Calculations: We get

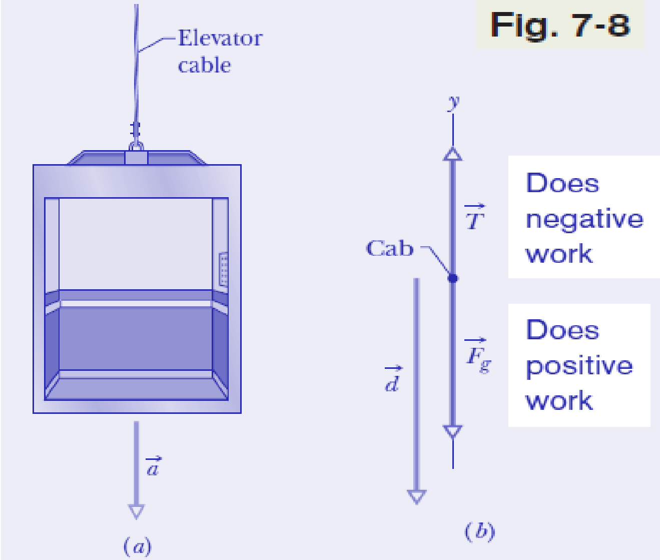
$$T - F_g = ma.$$

$$W_T = Td\cos\phi = m(a+g)d\cos\phi.$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ.} \qquad \text{(Answer)}$$

Fig. 7-8



(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

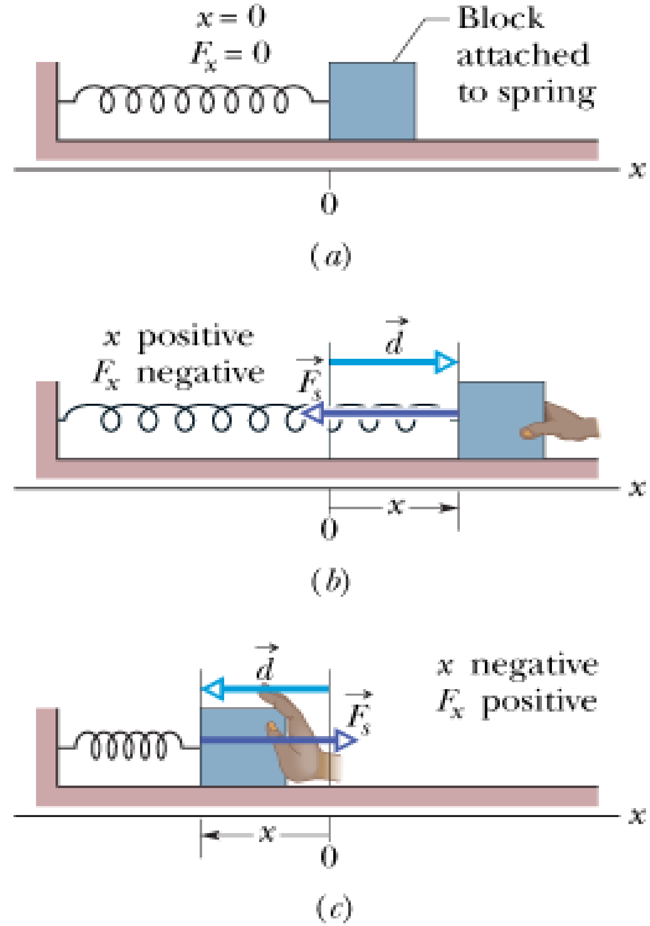
$$W = W_g + W_T = 5.88 \times 10^4\text{ J} - 4.70 \times 10^4\text{ J}$$
$$= 1.18 \times 10^4\text{ J} \approx 12\text{ kJ.} \qquad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\text{ kg})(4.0\text{ m/s})^2 + 1.18 \times 10^4\text{ J}$$
$$= 1.58 \times 10^4\text{ J} \approx 16\text{ kJ.} \qquad \text{(Answer)}$$

7.7: Work Done by a Spring Force



- **Hooke's Law:** To a good approximation for many springs, the force from a spring is proportional to the displacement of the free end from its position when the spring is in the relaxed state. The *spring force is given by:*

$$F_s = -kx$$

- The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant *k is called the* ***spring constant (or force constant) and is a measure*** of the stiffness of the spring.
- The net work W_s *done by a spring, when it has a distortion from* x_i *to* x_f*, is:*

$$W_s = \int_{x_i}^{x_f} -F_x \, dx.$$

$$W_s = \int_{x_i}^{x_f} -kx \, dx = -k \int_{x_i}^{x_f} x \, dx$$

$$= \left(-\tfrac{1}{2}k\right)[x^2]_{x_i}^{x_f} = \left(-\tfrac{1}{2}k\right)(x_f^2 - x_i^2).$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}$$

- ***Work W_s is positive if the block ends up closer to the relaxed position (x =0) than it was initially. It is negative if the block ends up farther away from x =0. It is zero if the block ends up at the same distance from x= 0.***

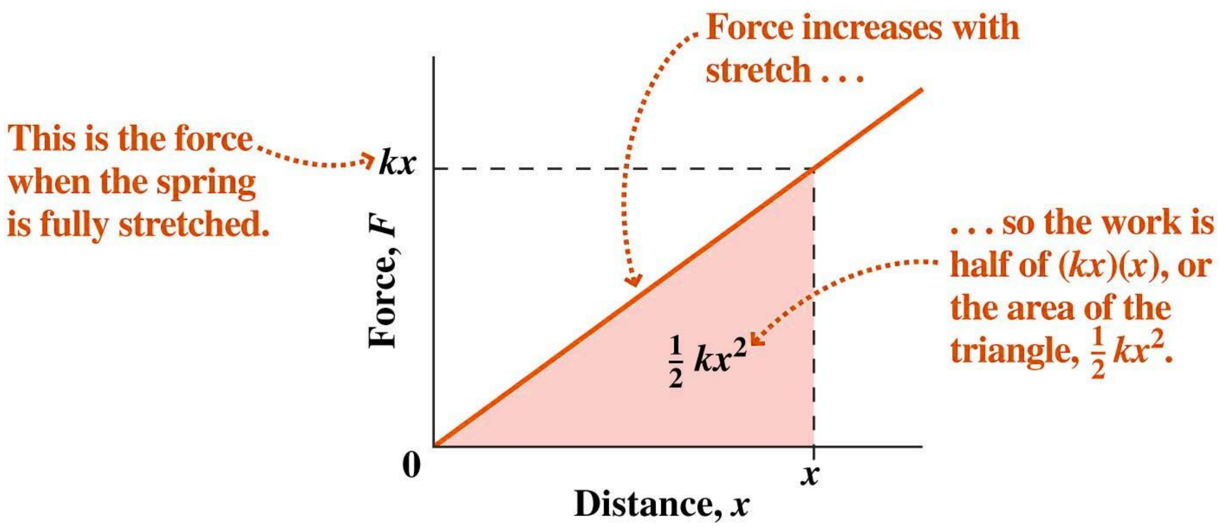
7.7: Work Done by a Spring Force

- A spring exerts a force: $F_s = -kx$

- Therefore the agent stretching a spring exerts a force $F = +kx$; and the work the agent does is:

$$W = \int_0^x F(x)\,dx = \int_0^x kx\,dx = \tfrac{1}{2}kx^2\Big|_0^x = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}kx^2$$

- In this case the work is the area under the triangular force-versus-position curve:



Example: Work done by spring

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

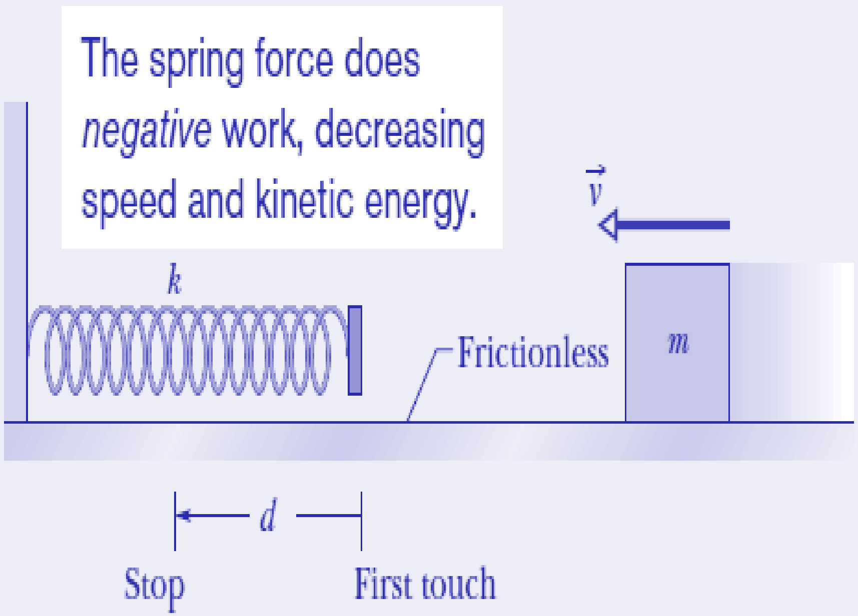


Fig. 7-10 A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50\ \mathrm{m/s})\sqrt{\frac{0.40\ \mathrm{kg}}{750\ \mathrm{N/m}}}$$

$$= 1.2 \times 10^{-2}\ \mathrm{m} = 1.2\ \mathrm{cm}. \qquad \text{(Answer)}$$

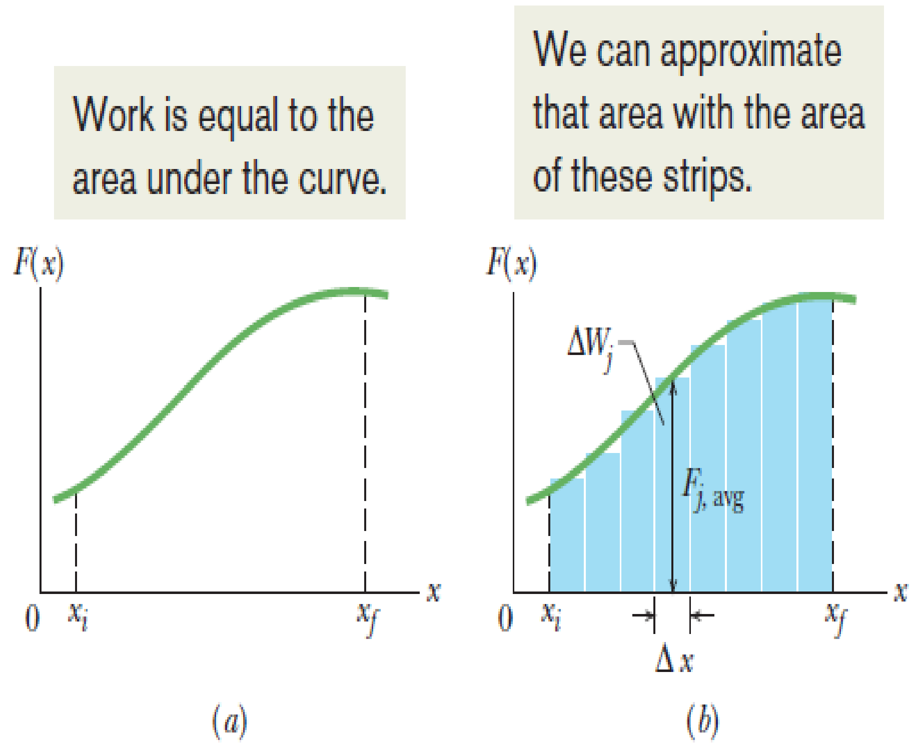
7.8: Work Done by a General Variable Force

- ***One-dimensional force, graphical analysis:***

- Divide the area under the curve of F(x) into a number of narrow strips of width x.
- Choose *x* small enough to permit us to take the force F(x) as being reasonably constant over that interval.
- Let $F_{j,avg}$ be the average value of F(x) within the jth interval.
- The work done by the force in the jth interval is approximately

$$\Delta W_j = F_{j,avg}\Delta x$$
$$\Rightarrow W = \sum \Delta W_j = \sum F_{j,avg}\Delta x$$

- W_j is then equal to the area of the jth rectangular, shaded strip.

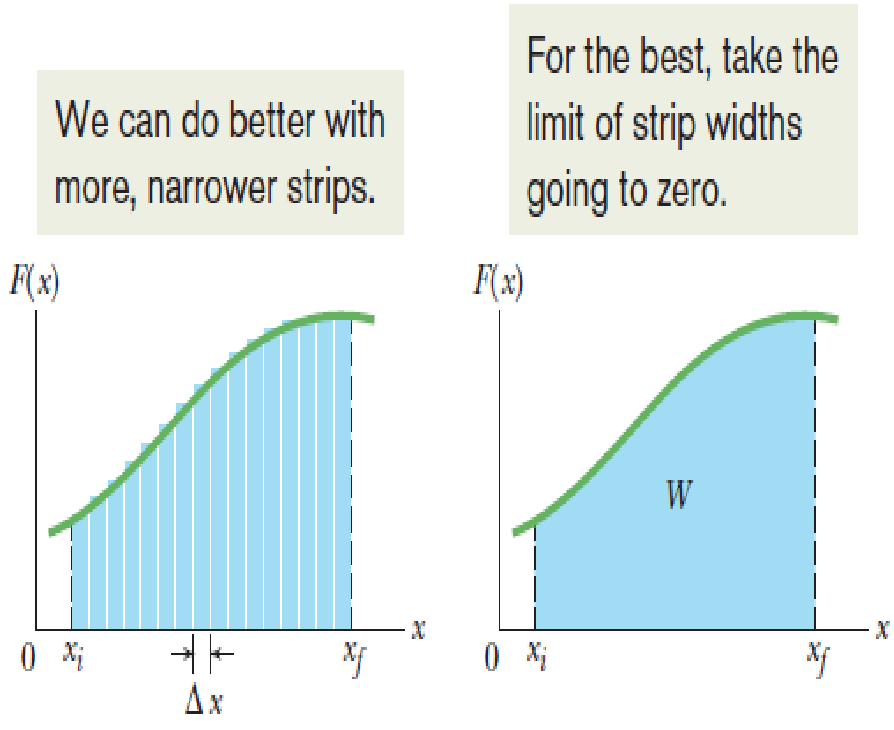


7.8: Work Done by a General Variable Force

- ***One-dimensional force, calculus analysis:***

  - We can make the approximation better by reducing the strip width Δx and using more strips (Fig. c).
  - In the limit, the strip width approaches zero, the number of strips then becomes infinitely large and we have, as an exact result.
  - Geometrically, the work is the area under the force vs. position curve.

$$\Delta W = \lim_{\Delta x \to 0} \sum F_{j,avg} \Delta x = \int_{x_i}^{x_f} F(x)\,dx$$



(c) (d)

7.8: Work Done by a General Variable Force (integration)

B. 1-D force, calculus analysis:

Integration *(anti-derivative)*

- The *definite integral* is the result of the limiting process in which the area is divided into ever smaller regions.

- Work as the integral of the force *F* over position *x* is written:

$$\boxed{W = \int_{x_1}^{x_2} F(x)\, dx}$$

- Integration is the opposite of differentiation, so integrals of simple functions are readily evaluated. For powers of *x*, the integral becomes

$$\boxed{\int_{x_1}^{x_2} x^n\, dx = \left.\frac{x^{n+1}}{n+1}\right|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}}$$

7.8: Work Done by a General Variable Force

- **3-D force:**

If

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k},$$

where F_x is the x-components of ***F*** and so on,

and

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}.$$

where *dx* is the *x*-component of the displacement vector ***dr*** and so on,

then

$$dW = \vec{F} \cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz.$$

Finally,

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

7.8: Work-Kinetic Energy Theorem with a Variable Force

A particle of mass *m* is moving along an x axis and acted on by a net force *F(x)* that is directed along that axis.

The work done on the particle by this force as the particle moves from position x_i to position x_f is:

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx,$$

But,

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v,$$

Therefore,

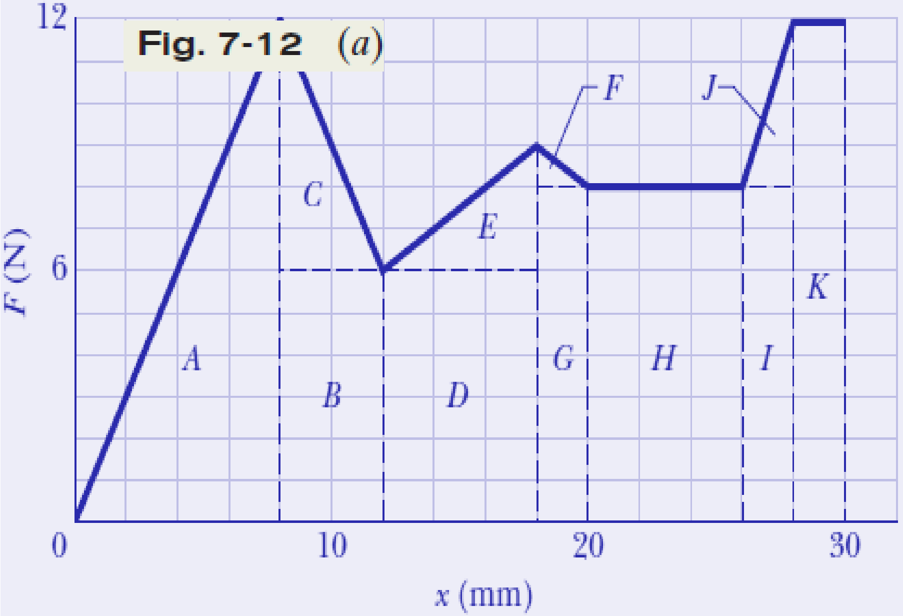
$$\boxed{W = \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.}$$

Example: Work calculated from graphical method

In an epidural procedure, as used in childbirth, a surgeon or an anesthetist must run a needle through the skin on the patient's back, through various tissue layers and into a narrow region called the epidural space that lies within the spinal canal surrounding the spinal cord. The needle is intended to deliver an anesthetic fluid. This tricky procedure requires much practice so that the doctor knows when the needle has reached the epidural space and not overshot it, a mistake that could result in serious complications.

The feel a doctor has for the needle's penetration is the variable force that must be applied to advance the needle through the tissues. Figure 7-12*a* is a graph of the force magnitude F versus displacement x of the needle tip in a typical epidural procedure. (The line segments have been straightened somewhat from the original data.) As x increases from 0, the skin resists the needle, but at $x = 8.0$ mm the force is finally great enough to pierce the skin, and then the required force decreases. Similarly, the needle finally pierces the interspinous ligament at $x = 18$ mm and the relatively tough ligamentum flavum at $x = 30$ mm. The needle then enters the epidural space (where it is to deliver the anesthetic fluid), and the force drops sharply. A new doctor must learn this pattern of force versus displacement to recognize when to stop pushing on the needle. (This is the pattern to be programmed into a virtual-reality simulation of an epidural procedure.) How much work W is done by the force exerted on the needle to get the needle to the epidural space at $x = 30$ mm?

Fig. 7-12 (*a*)



Calculations: Because our graph consists of straight-line segments, we can find the area by splitting the region below the curve into rectangular and triangular regions, as shown For example, the area in triangular region A is

$$\text{area}_A = \tfrac{1}{2}(0.0080\text{ m})(12\text{ N}) = 0.048\text{ N}\cdot\text{m} = 0.048\text{ J}.$$

Once we've calculated the areas for all the labeled regions in Fig. 7-12*b*, we find that the total work is

$$\begin{aligned} W &= (\text{sum of the areas of regions } A \text{ through } K) \\ &= 0.048 + 0.024 + 0.012 + 0.036 + 0.009 + 0.001 \\ &\quad + 0.016 + 0.048 + 0.016 + 0.004 + 0.024 \\ &= 0.238\text{ J}. \qquad \text{(Answer)} \end{aligned}$$

Example: Work from 2-D integration

Force $\vec{F} = (3x^2\ \text{N})\hat{\text{i}} + (4\ \text{N})\hat{\text{j}}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

Calculation: We set up two integrals, one along each axis:

$$W = \int_2^3 3x^2\,dx + \int_3^0 4\,dy = 3\int_2^3 x^2\,dx + 4\int_3^0 dy$$
$$= 3[\tfrac{1}{3}x^3]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3]$$
$$= 7.0\ \text{J}. \qquad \text{(Answer)}$$

The positive result means that energy is transferred to the particle by force $\vec{F}$. Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.

7.9: Power

The time rate at which work is done by a force is said to be the power due to the force. If a force does an amount of work *W* in an amount of time *t*, the average power due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \qquad \text{(average power).}$$

The instantaneous power *P* is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \qquad \text{(instantaneous power).}$$

The SI unit of power is the joule per second, or Watt (W).

In the British system, the unit of power is the footpound per second. Often the horsepower is used.

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W.}$$

7.9: Power

$$P = \frac{dW}{dt} = \frac{F \cos\phi \, dx}{dt} = F \cos\phi \left(\frac{dx}{dt}\right),$$

$$P = Fv \cos\phi.$$

$$P = \vec{F} \cdot \vec{v} \qquad \text{(instantaneous power).}$$

Example: Power, force, velocity

Figure 7-14 shows constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude 2.0 N; force $\vec{F}_2$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

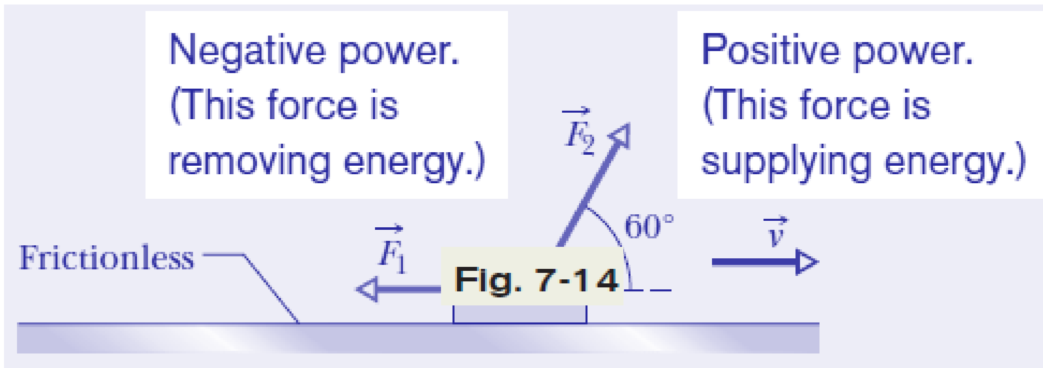


Fig. 7-14

Calculation: We use Eq. 7-47 for each force. For force $\vec{F}_1$, at angle $\phi_1 = 180°$ to velocity $\vec{v}$, we have

$$P_1 = F_1 v \cos \phi_1 = (2.0\ \text{N})(3.0\ \text{m/s}) \cos 180° = -6.0\ \text{W}. \quad \text{(Answer)}$$

This negative result tells us that force $\vec{F}_1$ is transferring energy *from* the box at the rate of 6.0 J/s.

For force $\vec{F}_2$, at angle $\phi_2 = 60°$ to velocity $\vec{v}$, we have

$$P_2 = F_2 v \cos \phi_2 = (4.0\ \text{N})(3.0\ \text{m/s}) \cos 60° = 6.0\ \text{W}. \quad \text{(Answer)}$$

This positive result tells us that force is transferring energy to the box at the rate of 6.0 J/s. The net power is the sum of the individual powers:

$$P_{net} = P_1 + P_2 = -6.0\ \text{W} + 6.0\ \text{W} = 0,$$

which means that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy of the box is not changing, and so the speed of the box will remain at 3.0 m/s. Therefore both P_1 and P_2 are constant and thus so is P_{net}.